INTERNATIONAL MONETARY FUND

Capital Holdup, Job Creation, and Skill Supply under Search Frictions

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WP/26/93

2026
MAY

WORKING PAPER

 WP/26/93

IMF Working Paper
Western Hemisphere Department

Capital Holdup, Job Creation, and Skill Supply under Search Frictions
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Authorized for distribution by Andrea Schaechter
May 2026

ABSTRACT: There has been renewed interest in revitalizing manufacturing, yet policy often confronts a challenge: firms hesitate to expand because they cannot reliably find suitably skilled workers (e.g., STEM-trained), while workers are reluctant to acquire those skills when jobs remain limited. This raises a policy question: intervene at the firm margin or the worker margin, or both? We study this question by extending Acemoglu and Shimer (1999) to a two-sector open-economy. The key friction is capital holdup: firms invest upfront to create jobs, but sunk investment weakens their wage bargaining positions, discouraging investment ex-ante. Because manufacturing is more capital intensive, holdup is more severe, leaving manufacturing employment inefficiently low. In the calibrated model, this inefficiency-induced industrial employment shortfall is about 1 percent of total employment – roughly one-fifth of the Latin America-East Asia gap. When the Hosios condition holds, the optimal policy can be solely on the firm side: an investment subsidy financed by an employment tax on firms. When the Hosios condition fails, an additional wedge distorting workers' sectoral choices emerges, and targeted training subsidies become welfare-improving.

JEL Classification Numbers:	E24, E60, J24, J64
Keywords:	Search and matching; employment
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* The authors would like to thank Gonzalo Garcia and seminar participants at the Central Bank of Chile, the IMF, and UC Davis for helpful comments. The views expressed in this paper are those of the authors and do not necessarily represent the views of the IMF, its Executive Board, or IMF Management.

1 Introduction

The development of the manufacturing sector has received renewed attention in recent years. This renewed focus reflects several factors, including historically faster productivity growth in manufacturing, spillovers through input-output linkages, its relevance for national security, and its role in the trade balances and employment.[1]

Yet policymakers often face a chicken-and-egg dilemma in developing the manufacturing sector. Employers report persistent difficulty filling manufacturing vacancies, citing a shortage of workers with STEM training.[2] At the same time, young people are less inclined to pursue STEM fields when manufacturing has a limited presence and offers few job opportunities.[3] This raises two related questions. First, should the government play a role—through taxes, subsidies, or other policies—to support its expansion? Second, if so, should public support target firms to incentivize more STEM-related job creation, or should it focus on encouraging STEM-trained labor supply?

This paper develops a Diamond–Mortensen-Pissarides (DMP) style search model with endogenous, job-specific capital in a two-sector small open economy to study these questions. Workers choose which sector to enter – manufacturing (m) or services (s) — subject to sector-specific training costs. Firms decide whether to create a vacancy and, if so, how much capital to invest. The two sectors differ in capital intensity, with a higher capital share in manufacturing than in services.

The key inefficiency is the familiar capital holdup problem emphasized in Grout (1984) and Acemoglu and Shimer (1999). Capital is installed upfront, before wage bargaining. This weakens firms' bargaining positions and discourages investment. Because manufacturing is more capital intensive, the distortion is more severe there. The contribution of this paper is to embed this same holdup problem in a two-sector setting with endogenous sectoral entry for both workers and firms.

Our first result is that in our two-sector environment, even when the Hosios (1990) condition is satisfied – so that search externalities are efficiently internalized – the decentralized equilibrium remains inefficient due to capital holdup. While the inefficiency from holdup is already well understood in a one-sector environment, what is new here is the

[1]See, for example, Duarte and Restuccia (2010) on faster productivity growth in manufacturing; Liu (2019) and McNerney et al. (2022) on propagation through input-output linkages; Hicks (2022) on national security considerations; and Autor et al. (2013) and Rodrik (2016) on trade and employment implications.

[2]For example, in 2023, the Semiconductor Industry Association reported a shortage of technician, engineering and computer science workforce in the industry and advocated for expanding the pipeline of STEM graduates.

[3]For example, EU's Eurobarometer 55.2 survey shows 42.5% respondents think that one of the main reasons for falling interests of youths in scientific studies and career is job and salary prospects.

implication for sectoral allocation of labor when there are two sectors. Because the manufacturing sector is more capital intensive, the holdup distortion depresses investment and job creation disproportionately in that sector. As a result, both the investment and sectoral composition of employment are inefficient in equilibrium. In contrast, conditional on efficient capital levels, workers' sectoral choice is efficient. This yields a sharp policy implication regarding the chicken-and-egg dilemma: when the Hosios condition holds, correcting underinvestment without subsidizing workers' training is sufficient to restore the planner's allocation. The associated optimal policy is fiscally neutral: an investment subsidy is fully financed by a lump-sum fee on firms in proportion to employment.

Our second result is that when the Hosios (1990) condition fails, an additional wedge distorts workers' sectoral choice, so that correcting underinvestment alone is no longer sufficient to replicate the planner's allocation. In particular, in the empirically relevant case where workers' bargaining power is smaller than the elasticity of matching with respect to unemployment, and the training cost of entering manufacturing is higher than services, it becomes harder for manufacturing workers to obtain sufficient surplus to offset their higher training costs. As a result, the optimal policy generally combines investment subsidies on firms with targeted training subsidies on workers.

Finally, the open economy setting allows us to study how trade interacts with manufacturing development. Consider a country with comparative advantage in manufacturing, so that it exports goods and imports services: Greater openness to services imports lowers the relative prices of services and, in turn, raises the profitability and employment of manufacturing relative to services. In addition, when home production is denominated in either final consumption or services alone, unemployment declines, because the value of home production relative to manufacturing employment income becomes lower.

1.1 Relations to the Literature

Our paper contributes to the macroeconomic literature on the sectoral shares of the economy. Past studies in the area of structural change, such as Duarte and Restuccia (2010), Uy et al. (2013), and Huneeus and Rogerson (2024), examine how sectoral productivity differences and trade account for cross-country or -time variation in the shares of manufacturing and services. Because their goal is to explain these patterns positively, much of this literature relies on competitive environments in which the equilibrium is efficient, leaving limited scope for policy design. In contrast, we study manufacturing development in a frictional labor market where capital is subject to holdup and demonstrate the inefficiencies of the equilibrium, a rationale for policy intervention. A few recent exceptions incorporate search frictions into structural change models, for example, Feng et al. (2024). Relative to that work, our model features endogenous capital, trade openness, and an explicit focus on efficiency, allowing us to characterize whether policy should prioritize firm-side investment or worker-side training.

Our paper also complements the growing literature on the rationale and design of industrial policies to support certain industries, including the classic arguments based on coordination failure under increasing-return-to-scale (Matsuyama (1991)) and the role of industries within production networks (Liu (2019)). We emphasize a different mechanism – capital holdup interacting with search frictions and endogenous skill choice – in an otherwise standard constant-return-to-scale environment. This channel links industrial policy directly to both job creation and skill acquisition, yielding sharp conditions under which investment subsidies alone are sufficient versus when training subsidies are warranted.

From a labor-search perspective, our model builds on the holdup problem in wage bargaining highlighted in Acemoglu and Shimer (1999) but extends it to a two-sector, open-economy setting with endogenous education choices. Closely-related work, such as Cardullo et al. (2015), develops a two-sector closed-economy search model and shows how cross-industry differences in holdup help explain cross-industry investment growth. Our focus is on the inefficiencies implied by holdup and search frictions and how they map into policy prescriptions, and how they depend on whether the Hosios condition holds. Our analysis also relates to the classic efficiency results in search-and-matching models. In the standard one-sector DMP framework, deviations from the Hosios condition create a wedge between private incentives and the social contribution to match formation, typically showing up as inefficient vacancy creation and market tightness (Hosios (1990); Mortensen and Pissarides (1994); Pissarides (2000)). Related work shows that the same misalignment can also distort match-related investments, including schooling choices in Flinn and Mullins (2015) and firms' training incentives under wage compression in Acemoglu and Pischke (1999), and that when match output depends on market tightness (Mangin and Julien (2021)). Our contribution is to show that in a two-sector environment with segmented labor markets and costly ex ante training, deviations from Hosios condition generate an additional distortion on the worker's sectoral choice margin. In equilibrium, workers' sectoral choice is pinned down by their privately captured share of match surplus, while the planner's condition depends on the matching elasticity. This creates a novel sectoral-choice wedge even conditional on efficient vacancy creation and efficient capital levels.

Finally, our paper is related to the studies on the field-of-study choice and economic development, such as Maloney and Valencia Caicedo (2022) and Murphy et al. (1991). Maloney and Valencia Caicedo (2022) show that the density of engineers in 1900 helps explain today's income variation across U.S. counties. Their explanation is that engineers facilitate technology adoption. Murphy et al. (1991) find that countries with more engineering and fewer law college graduates experienced faster income growth. They attribute the field-of-study choice to whether an economy encourages more entrepreneurship or rent-seeking. We complement these studies by embedding education choice in a frictional labor market where investment and job creation respond endogenously to policy and trade, and by studying how labor market frictions distort workers' field-of-study choices.

The rest of the paper is organized as follows. Section 2 presents the stylized facts linking sectoral employment and the share of STEM graduates. Section 3 outlines the model and characterizes the decentralized equilibrium. Section 4 defines and characterizes the social planner's problem. Section 5 discusses the efficiency of decentralized equilibrium. Section 6 and 7 cover calibration and quantitative results. Section 8 concludes.

2 Stylized Facts

This section documents two stylized facts: (i) substantial cross-country variation in the employment share of the industrial sector, and (ii) substantial variation in the share of STEM graduates among all tertiary graduates. We then show that countries with higher STEM shares tend to have a larger industrial employment share.

Industrial employment. We measure sectoral employment using data from Groningen Growth and Development Centre (GGDC). We combine two sources. The GGDC Economic Transformation Database (ETD), released in 2021, provides annual sectoral employment and value added for 12 sectors from 1990 to 2018. However, the ETD mainly covers low- and middle-income economies, with limited coverage of high-income economies. To broaden country coverage, we merge the ETD sample with the GGDC 10-Sector Database released in 2014, which reports annual sectoral employment and value-added for most high-income economies and selected middle-income economies from 1950 to 2011. The combined sample includes 60 economies.

Following the structural change literature (e.g., Uy et al. (2013), Huneeus and Rogerson (2024)), we aggregate sectors into agriculture, industry, and services. The industrial sector includes mining, manufacturing, utilities, and construction, while the service sector comprises all remaining sectors outside agriculture and industry. For each country, we calculate sectoral employment shares using the most recent two years available: 2017-18 for economies covered by the ETD and 2010-11 for countries that appear only in the GGDC 10-Sector Database.

Industry employment shares vary widely across countries, consistent with previous studies (e.g., Rodrik (2016), Huneeus and Rogerson (2024)). In our sample, the industrial employment share ranges from 4 percent in Mozambique to 33 percent in China. This heterogeneity persists even among economies with comparable income levels. For example, with the exception of Mexico, Latin American countries such as Peru, Chile, Colombia, and Brazil have industrial employment shares between 16 and 19 percent, while East Asian countries have noticeably higher shares – about 25 percent in Korea and 33 percent in China.

STEM graduate share. We measure field-of-study composition using data from the UNESCO Institute for Statistics (UIS), which reports the distribution of tertiary graduates across fields. UIS coverage begins in 2000, though for many middle- and low-income countries it starts only in the 2010s. We define the STEM share as the sum of graduates in natural sciences, mathematics and statistics, information and communication technologies, and engineering, manufacturing and construction programs. For countries with data available for multiple years, we use the earliest available year so that for most countries, STEM shares are measured before industrial employment shares. This yields a sample of 147 economies, with their STEM shares ranging from 2 percent in Namibia (2001) to 45 percent in Malaysia (2004). China and Japan are not in the UIS database, so for these two countries we supplement the data using statistics from the Chinese Ministry of Education's Educational Statistics and the OECD Statistics.

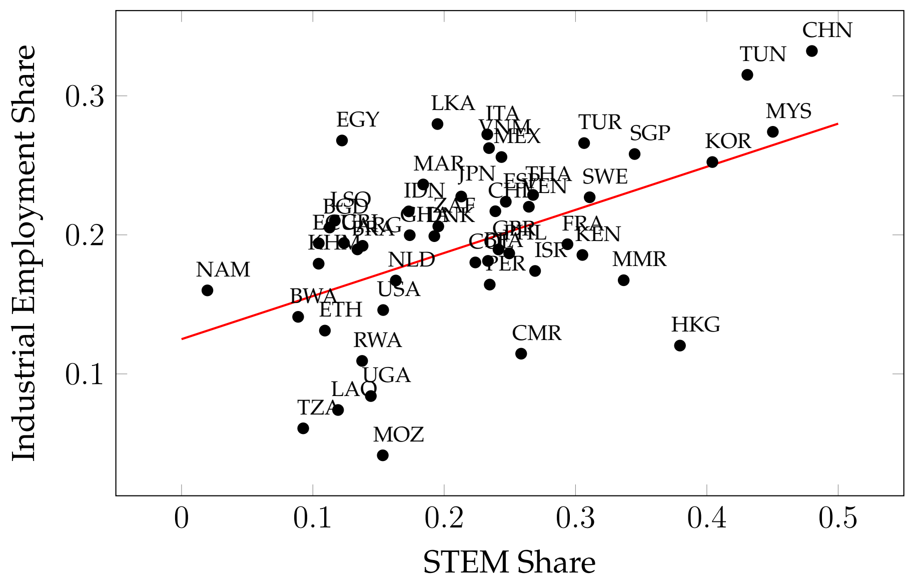
2.1 Relationship between industrial employment and STEM share

Figure 1 shows that economies with higher industrial employment shares also tend to have higher STEM shares. To quantify this relationship, we estimate a cross-country OLS regression

$$S_i = \beta_{stem} STEM_i + \delta \cdot X_i + \epsilon_i \tag{1}$$

where S_i is the industrial employment share in country i, $STEM_i$ is the STEM graduate share, and X_i includes controls commonly used in the structural change literature: PPP-adjusted log GDP per capita, the agricultural employment share, and educational attainment. GDP per capita comes from Penn World Table 10.01. Educational attainment is measured as the share of adults aged 25 or older who have completed short-cycle tertiary education or higher, also from UNESCO-UIS.

Figure 1. STEM share vs. industrial employment share.



Note: STEM share is calculated as the annual number of tertiary graduates in STEM fields divided by total number of tertiary graduates. Industrial employment share is calculated as the number of employees in the industry sector divided by total employment.

The estimates in Table 1 show a positive and statistically significant correlation between STEM graduate shares and industrial employment shares. In columns (1) and (3), β_{STEM_i} is significant at the 1 percent level: a 1 percentage point increase in STEM share is associated with a 0.24-0.30 percentage point rise in industrial employment share. The coefficient on overall educational attainment is small and negative, suggesting that variation in the composition of tertiary graduates is more closely associated with industrial employment than educational attainment alone.

Correlation and causality. As will become clearer in the next section, our model is built on the premise that sectoral specialization and skill supply are jointly determined: countries with a larger industrial sector tend to attract more talent into STEM fields, which we use as a proxy for industrial sector-related skill supply, while a stronger STEM talent pipeline may support a larger industrial sector. Therefore, the purpose of Figure 1 and Equation (1) is to document this relationship as a correlation, rather than to establish a causality. That said, we take a modest step towards exploring the direction of the relationship by re-estimating Equation (1) using only the subsample of countries for which STEM share data are available at least 5 years prior to the employment data. As presented in Column (2), the coefficient on $STEM_i$ remains significant at the 1 percent level. While this exercise does not resolve concerns about omitted variables or endogeneity, it suggests that the positive association is not driven solely by contemporaneous correlation and is consistent with a causal interpretation.

Table 1: Dependent Variable: Industrial Employment Share

	(1) All	(2) Subsample	(3) All
STEM share ($STEM_i$)	0.24***	0.19**	0.30***
	(0.07)	(0.08)	(0.08)
Edu. attainment	-0.001***	-0.000**	-0.001
	(0.00)	(0.00)	(0.00)
Agricultural emp. share	-0.20***	-0.18***	–
	(0.04)	(0.05)	
Log income	–	–	0.02***
			(0.01)
Constant	0.23***	0.21***	-0.03
	(0.03)	(0.03)	(0.07)
Observations	50	31	50
R^2	0.60	0.52	0.34

Note: The ”Sub Sample” includes only economies with STEM share data available at least 5 years prior to the employment share data. Robust standard errors in parentheses. $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

3 Model

3.1 Environment and timing

We study a small open economy with two sectors: manufacturing (m) and services (s). Time is continuous and discounted at rate $r > 0$. A unit mass of risk-neutral workers and a continuum of risk-neutral firms populate the economy. Workers choose sector-specific educational tracks and then search for jobs in the corresponding sector. In each sector, jobs are formed with DMP-type search and matching frictions. Let L_m and L_s denote the masses of workers who have acquired the skills relevant for sectors m and s, respectively. Thus, L_i is the measure of the labor force in sector i. The total labor force in the two sectors is fixed at L: $L = L_m + L_s$, while the allocation across sectors is endogenous. Let unemployment within sector i be $u_i \equiv L_i - n_i$, where n_i stands for employment in sector i.

3.2 Preferences, prices, and trade.

Employed workers consume a composite of manufacturing and service goods:[4]

$$c^E = (c_m^E)^\alpha (c_s^E)^{1-\alpha}. \tag{2}$$

Unemployed workers receive a flow payoff z, which can be interpreted as the value of nonmarketable home production.

Aggregation. Let n denote total manufacturing and service employment. Let C_i denote aggregate consumption of good i. Aggregate consumption satisfies

$$\begin{aligned} nc_m^E &\equiv C_m, \\ nc_s^E &\equiv C_s, , \\ nc^E &= n(c_m^E)^\alpha (c_s^E)^{1-\alpha} \\ &= C_m^\alpha C_s^{1-\alpha}. \end{aligned}$$

Hence, we can define the aggregate final consumption as

$$C \equiv C_m^\alpha C_s^{1-\alpha}. \tag{3}$$

We therefore only need to keep track of aggregate consumption C_m, C_s and C in equilibrium. The unit price of final consumption is:

$$P = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} P_m^\alpha P_s^{1-\alpha}. \tag{4}$$

Manufacturing goods are fully tradable and are perfect substitutes for foreign-produced manufacturing goods. Hence, the price of domestic manufacturing goods equals the world price : $P_m = p_m = p_m^F$. We set $p_m^F = 1$ as the numeraire. The service composite C_s is a CES aggregate of domestically produced C_s^H and imported services C_s^F with elasticity of substitution ν:

$$C_s = \left[\chi (C_s^H)^{\frac{\nu-1}{\nu}} + (1-\chi)\,(C_s^F)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \quad \chi \in (0,1). \tag{5}$$

[4] Here we assume individual workers are grouped into households, and employed members within the same household can pool their consumption, so that every employed member's consumption level would be identical (c^E). This assumption helps us avoid distinguishing between the consumption of workers having manufacturing jobs and of workers with services jobs.

where $1-\chi$ governs the service-trade openness , and $\nu > 1$ implies that foreign and domestic services are gross substitutes. The import price of services is exogenous at p_s^F. The domestic service price p_s is endogenous in equilibrium. The unit price of the service composite is:

$$P_s = \left[\chi\, p_s^{1-\nu} + (1-\chi)\,(p_s^F)^{1-\nu}\right]^{\frac{1}{1-\nu}}, \tag{6}$$

Trade pattern. We assume that the home country has a comparative advantage in manufacturing relative to the rest of the world: $p_s/p_m > p_s^F/p_m^F = 1$. Therefore, the economy is a net exporter of manufacturing goods and a net importer of services. We assume balanced trade in every period.

Consumption demand. Let total nominal expenditure be $E \equiv PC$. Standard utility maximization implies the following consumption demands:

$$C_m = \frac{\alpha}{p_m} E, \tag{7}$$

$$C_s = \frac{1-\alpha}{P_s} E, \tag{8}$$

$$C_s^H = \chi \left(\frac{p_s}{P_s}\right)^{-\nu} C_s, \tag{9}$$

$$C_s^F = (1-\chi) \left(\frac{p_s^F}{P_s}\right)^{-\nu} C_s. \tag{10}$$

3.3 Labor market and production

Labor markets in manufacturing and services are segmented. Each sector $i \in \{m, s\}$ operates a labor market with DMP-style search and matching frictions. Each job consists of a one-vacancy-one-worker pair. Let u_i and v_i denote the masses of unemployed workers and unfilled vacancies in sector i. Matches are formed according to

$$m_i = M(u_i, v_i). \tag{11}$$

The matching function $M(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one. In the quantitative exercise section, we assume

$$M(u_i, v_i) = u_i^\eta v_i^{1-\eta}, \tag{12}$$

where $\eta \in (0,1)$ is the elasticity of matches with respect to unemployment. Define market tightness as

$$\theta_i \equiv v_i/u_i. \tag{13}$$

The vacancy–filling rate is then

$$q_i(\theta_i) \equiv M_i(u_i, v_i)/v_i, \tag{14}$$

and the job–finding rate is

$$\theta_i q_i(\theta_i) \equiv M_i(u_i, v_i)/u_i. \tag{15}$$

Job-specific capital and production. Creating a vacancy in sector i requires installing job–specific capital $k_i \geq 0$. Once chosen, the capital level cannot be changed until the capital depreciates and the vacancy is destroyed. Because capital is sunk at the time of wage bargaining, it generates a holdup problem. Once a vacancy is filled, the match produces output at rate $f_i(k_i)$ with $f_i' > 0$, $f_i'' < 0$, and $f_i(0) = 0$. Total output in sector i is thus the product of per-match output and employment:

$$Y_i = f_i(k_i)n_i, \qquad i \in \{m, s\}. \tag{16}$$

Following Acemoglu and Shimer (1999), we assume that the job-specific capital attached to any vacancy, filled or unfilled, is lost with exogenous Poisson probability s_i, at which point the match is also separated.[5]

Employment and unemployment dynamics. Employment evolves according to:

$$\dot{n}_i = m_i - s_i n_i, \qquad i \in \{m, s\}. \tag{17}$$

[5]Allowing s_i to differ by sector helps reconcile two empirical regularities: (i) manufacturing and services exhibit similar or slightly lower *unemployment rates* in manufacturing, $\bar{u}_m \lesssim \bar{u}_s$, where $\bar{u}_i \equiv u_i/L_i$; (ii) manufacturing is more capital intensive. Higher k_m raises hold–up costs and would otherwise push $\bar{u}_m$ above $\bar{u}_s$; taking $s_m < s_s$ captures longer job durations in capital–intensive matches and restores consistency with the data without inflating education costs or wages.

Within each sector,

$$u_i = L_i - n_i, \tag{18}$$

$$v_i = \theta_i u_i. \tag{19}$$

Let x_i denote the flow of new vacancies created, vacancy dynamics thus satisfy

$$\dot{v}_i = x_i - s_i v_i - m_i. \tag{20}$$

Unemployment dynamics are given by

$$\dot{u}_i = s_i(L_i - u_i) - \theta_i q_i(\theta_i) u_i. \qquad i \in \{m, s\},$$

In steady state, the unemployment *rate* $\bar{u}_i \equiv u_i / L_i$ is given by

$$u_i^{ss} = \frac{s_i}{s_i + \theta_i q_i(\theta_i)} L_i, \qquad \bar{u}_i^{ss} = \frac{u_i^{ss}}{L_i} = \frac{s_i}{s_i + \theta_i q_i(\theta_i)}. \tag{21}$$

Education and workers' sectoral choices. Before entering the labor market, workers choose which sector-specific skills to acquire and this choice is permanent. Acquiring the skills for sector i requires a one-time training cost ϵ_i, measured in units of the final consumption good. We assume $\epsilon_m \geq \epsilon_s$, reflecting the idea that manufacturing-related skill training is typically more costly than service-related skill training.

3.4 Bellman equations and definition of decentralized equilibrium

Let J_i^U and $J_i^E(k_i)$ denote a worker's values of unemployment and employment in sector i, and let $J_i^V(k_i)$ and $J_i^F(k_i)$ denote a firm's values of a vacancy and a filled job. Here, $J_i^V(k_i)$ is the value of a vacancy before netting out the capital installation cost. A firm that creates a vacancy with capital k_i therefore earns a net value $J_i^V(k_i) - P_k k_i / P$ measured in units of the final consumption good. The corresponding Bellman equations are:

$$
\begin{aligned}
rJ_i^U &= z + \theta_i q_i(\theta_i)\big(J_i^E(k_i^*) - J_i^U\big), && (22)\\
rJ_i^E(k_i) &= \frac{w_i(k_i)}{P} + s_i\big(J_i^U - J_i^E(k_i)\big), && (23)\\
rJ_i^V(k_i) &= q_i(\theta_i)\big(J_i^F(k_i) - J_i^V(k_i)\big) - s_i J_i^V(k_i), && (24)\\
rJ_i^F(k_i) &= \frac{p_i f_i(k_i) - w_i(k_i)}{P} - s_i J_i^F(k_i), && (25)\\
\text{where } k_i^* &= \arg\max_{k_i} \left\{ J_i^V(k_i) - \frac{P_k\, k_i}{P} \right\}. && (26)
\end{aligned}
$$

Equation (22) gives the flow value of an unemployed worker in sector i: the unemployment income plus the expected payoff from finding a job. We model unemployment income as home production of z, measured in units of final consumption good. The expected gain from finding a job equals the job finding rate $\theta_i q_i(\theta_i)$ times the gain from employment, $J_i^E(k^*) - J_i^U$ where k_i^* denotes the equilibrium level of capital per worker in sector i. Equation (23) gives the flow value of an employed worker in sector i conditional on the job's capital level k_i, taking into account the real wage $w_i(k_i)/P$ and the separation rate s_i. Equation (24) gives the flow value of a vacancy with capital level k_i, which includes the expected gain from the vacancy being filled and the expected loss from the destruction of the vacancy. The level of k_i is optimally chosen by firms to maximize the value of the vacancy. Finally, (25) gives the flow value of a filled job with capital k_i: the firm receives real output $p_i f_i(k_i)/P$, pays labor $w_i(k_i)/P$, and accounts for the separation rate s_i.

Capital choice (holdup). The firm chooses the capital level when the vacancy is created. Once the vacancy is created, the capital cost becomes sunk. The firm therefore chooses capital to maximize its expected profit $J_i^V(k_i) - \frac{P_k k_i}{P}$.

Wage bargaining. After a worker is matched with a vacancy, the wage is determined through Nash bargaining, with the worker's bargaining weight given by $\beta \in (0,1)$:

$$(1-\beta)\big(J_i^E(k_i) - J_i^U\big) = \beta\big(J_i^F(k_i) - J_i^V\big). \tag{27}$$

Free entry condition. Free entry by firms implies that the expected equilibrium profit from creating a vacancy is zero:

$$J_i^V(k_i^*) = \frac{P_k k_i^*}{P}. \tag{28}$$

Workers' sectoral choice. In equilibrium, workers are indifferent between entering the manufacturing or service markets:

$$J_m^U - J_s^U = \epsilon_m - \epsilon_s \tag{29}$$

Definition of decentralized equilibrium. A stationary decentralized equilibrium (DE) consists of

(i) Prices (p_s, P_s, P),

(ii) Sectoral labor market objects $\{L_i, \theta_i, u_i, n_i, v_i, k_i, w_i\}_{i \in \{m,s\}}$, and

(iii) Good market quantities $\{E, C_m, C_s, C_s^H, C_s^F, X_m, M_s\}$,

Such that:

1. Given prices (p_m, p_s, p_s^F) and total expenditure E, consumption demands satisfy (7) - (10).

2. Firms and workers optimize in the labor market, so that the value functions satisfy (22)- (26) and wages $w_i(k_i)$ satisfy Nash bargaining (27).

3. Free entry of firms characterized by (28).

4. Workers' sectoral choices satisfy (29).

5. Total labor supply satisfies $L_s + L_m = L$.

6. Production satisfies $Y_i = f_i(k_i)n_i$.

7. Good markets are cleared and trade is balanced, as in (30) - (34), and

8. Labor market flows satisfy (21).

3.5 Characterization of the Decentralized Equilibrium

We characterize the stationary decentralized equilibrium in two blocks.

- A *goods market block* maps (Y_m, Y_s, I) into the equilibrium domestic service price p_s consistent with household utility maximization, market clearing, and balanced trade. This mapping is independent of the structure of labor market.

- A *labor market block* maps p_s into labor market outcomes $(\theta_i, k_i, w_i, n_i, L_i)$, and therefore into (Y_m, Y_s, I).

  The decentralized equilibrium is a fixed point of these two mappings.

3.5.1 Goods market equilibrium block

This block derives the equilibrium domestic service price p_s, as well as consumption and trade flows, given foreign prices (p_m^F, p_s^F), real output (Y_m, Y_s), and nominal investment I.

Goods-market clearing and balanced trade. In the market for manufacturing goods, total output must equal the sum of domestic consumption, investment, and exports:

$$Y_m = C_m + I + X_m. \tag{30}$$

Total investment is given by

$$I = P_k(x_m k_m + x_s k_s), \tag{31}$$

where x_i denotes the flow of newly created vacancies in sector i.[6]

For services,

$$C_s^H = Y_s, \tag{32}$$

$$C_s^F = M_s. \tag{33}$$

Trade is balanced:

$$p_m X_m = p_s^F M_s. \tag{34}$$

Nominal GDP is defined as

$$Y^N = p_m Y_m + p_s Y_s. \tag{35}$$

Total nominal consumption expenditure

$$E \equiv PC \equiv p_m C_m + p_s C_s^H + p_s^F M_s \tag{36}$$

$$= p_m(Y_m - I) + p_s Y_s, \tag{37}$$

where the last equality uses (30) - (34).

Proposition 1. *Given exogenous prices p_m, p_s^F, output (Y_m, Y_s), and investment I, the domestic service price $p_s(p_m, p_s^F, Y_m, Y_s, I)$ is determined by*

$$Y_s = \chi(1-\alpha)\left(p_m(Y_m - I) + p_s Y_s\right) p_s^{-\nu}\, P_s^{\nu-1}, \tag{38}$$

where P_s is given by (6).

Given p_s, the final-good price P and nominal consumption E are determined by (4) and (37). Consumption allocations are determined by (7)-(10). Exports X_m and imports M_s are pinned down by (33) and (30).

[6] At steady state, the number of vacancies created in each moment should equal the number of vacancies destroyed: $x_i = s_i(n_i + v_i)$.

3.5.2 Labor market equilibrium block

From (24) and (25), we have

$$J_i^F(k_i) = \frac{p_i f(k_i) - w_i(k_i)}{P(r+s_i)}, \tag{39}$$

$$J_i^V(k_i) = \frac{q_i(\theta_i)}{r+s_i+q_i(\theta_i)} J_i^F(k_i). \tag{40}$$

A firm chooses k_i to maximize its expected profit $J_i^V(k_i) - P_k k_i/P$. The first-order condition is therefore

$$P_k = \frac{q_i(\theta_i)}{r+s_i+q_i(\theta_i)} \frac{p_i f'(k_i) - w_i'(k_i)}{r+s_i}. \tag{41}$$

From (23) and (24) , we have

$$\frac{p_i f(k_i) - w_i(k_i)}{P} = \frac{(1-\beta_i)\big(r+s_i+q_i(\theta_i)\big)}{r+s_i+(1-\beta_i)q_i(\theta_i)} \big[\frac{p_i f(k_i)}{P} - rJ_i^U\big]. \tag{42}$$

Substitute it into (41), we have the optimality condition for k_i:

$$P_k = \frac{q_i(\theta_i)(1-\beta_i)}{r+s_i} \frac{p_i f'(k_i)}{r+s_i+(1-\beta_i)q_i(\theta_i)}.$$

Total surplus $S_i(k_i)$: To build intuition, it is useful to define the total surplus of a match in sector i as

$$S_i(k_i) \equiv (J_i^E(k_i) - J_i^U) + (J_i^F(k_i) - J_i^V(k_i)).$$

Nash bargaining in (27) then implies

$$J_i^E - J_i^U = \beta S_i, \quad J_i^F - J_i^V = (1-\beta)S_i. \tag{43}$$

Using (22)–(25), we obtain:

$$r(J_i^E(k_i) - J_i^U) = \frac{w_i(k_i)}{P} - z - (s_i + \theta_i q(\theta_i))(J_i^E(k_i) - J_i^U), \tag{44}$$

$$r(J_i^F(k_i) - J_i^V(k_i)) = \frac{p_i f_i(k_i) - w_i(k_i)}{P} - (s_i + q(\theta_i))(J_i^F(k_i) - J_i^V(k_i)). \tag{45}$$

Adding these two equations and using (43), we get

$$S_i(k_i) \;=\; \frac{p_i f_i(k_i)/P - z}{D_i(\theta_i)}, \tag{46}$$

where

$$D_i(\theta_i) \;\equiv\; r + s_i + (1-\beta)\, q_i(\theta_i) + \beta\, \theta_i q_i(\theta_i). \tag{47}$$

Thus, the total surplus of match in sector i equals the output flow net of home production, discounted by D_i. The discount term D_i reflects the time discount rate r, the separation rate s_i, and the adjustment due to the transitions between vacancies and matches and between unemployment and employment. Note that the sunk capital cost $P_k k_i$ does not enter $S_i(k_i)$ because capital investment happens before wage bargaining.

The other two key equilibrium conditions are the free-entry condition (28) and the workers' sectoral choice condition (29). Given total match surplus $S_i(k_i)$, the value of a vacancy $J_i^V(k_i)$ and the value of unemployment J_i^U follow directly from (22), (24) and Nash bargaining:

$$J_i^V(k_i) = \frac{q_i(\theta_i)(J^F(k_i) - J_i^V(k_i))}{r+s_i} = \frac{q_i(\theta_i)(1-\beta)S_i(k_i)}{r+s_i}, \tag{48}$$

$$J_i^U = \frac{\theta_i q_i}{r}(J_i^E(k_i^*) - J_i^U) = \frac{\beta\theta_i q_i}{r} S_i(k_i^*). \tag{49}$$

Proposition 2 (Decentralized equilibrium)**.** *In steady state, the decentralized equilibrium* $(k_i^*, \theta_i^*, L_s^*)$ *is characterized by the following conditions:*

$$\{\textit{Capital choice}\} : P_k = \frac{q_i(\theta_i)(1-\beta)}{r+s_i}\,\frac{p_i f'(k_i)}{r+s_i+(1-\beta)q_i(\theta_i)}, \qquad i \in \{m, s\}; \tag{50}$$

$$\{\textit{Tightness}\} : \frac{P_k\, k_i}{P} = \frac{q_i(\theta_i)(1-\beta)}{r+s_i}\,\frac{p_i f_i(k_i)/P - z}{D_i(\theta_i)}, \qquad i \in \{m, s\}; \tag{51}$$

$$\{\textit{Sectoral choice}\} : r(\epsilon_m - \epsilon_s) = \frac{\beta\,\theta_m q_m(\theta_m)}{D_m(\theta_m)}\,[p_m f_m(k_m)/P - z] - \frac{\beta\,\theta_s q_s(\theta_s)}{D_s(\theta_s)}\,[p_s f_s(k_s)/P - z], \tag{52}$$

Remarks. Equation (50) determines firms' optimal capital choices. The left-hand side is the rental cost of capital required to create or maintain a vacancy. The right-hand side captures the marginal life-time return from additional investment. The term $\frac{p_i f_i'(k_i)}{r+s_i+(1-\beta_i)q_i(\theta_i)}$ captures the marginal increase in surplus from higher capital.[7] This flow return is dis-

[7]The denominator $r + s_i + (1-\beta_i)q_i$ differs from D_i in (46) and (47). The derivation of (50) takes J_i^U as given because it is an equilibrium outcome. The derivation of (46) and (47) takes into account that a change k_i can also affect J_i^U in equilibrium.

counted by interest rate and separation rate, $(r + s_i)$, and is adjusted for the vacancy filling rate, $q_i(\theta_i)$. Importantly, the $(1 - \beta)$ term captures the *capital holdup problem* highlighted in Acemoglu and Shimer (1999) and Cardullo et al. (2015): firms internalize only a fraction of the match surplus when they choose k_i, leading to under–investment relative to the social planner's allocation which we characterize in the next section.

Equation (51) reflects the free-entry condition in equilibrium. A firm creates a vacancy at cost $P_k k_i/P$, and its return is $1 - \beta$ share of total surplus flow. This surplus flow is converted to a life-time value by discounting at $r + s_i$ and adjusting for the vacancy filling rate.

Equation (52) governs workers' sectoral choices. The left-hand side is the training cost differential, while the right-hand side captures the difference in workers' expected surpluses from entering each sector. The expected surplus from entering sector i is given by the worker's bargaining share β, times the job-finding rate $\theta_i q_i(\theta_i)$ and total surplus S_i.

4 Social Planner's (SP) Problem

Let $\mathcal{C}(Y_m, Y_s, I)$ denote the level of final consumption implied by the goods-market equilibrium for given output $(Y_m Y_s)$ and investment I, as characterized by Proposition 1. The social planner chooses $\{\theta_i(t), k_i(t), L_s(t)\}_{t\geq 0}$ to maximize the present discounted value of output net of capital and education costs:[8]

$$\max_{\{\theta_i, k_i, L_s\}} \int_0^\infty \Big\{\mathcal{C}(Y_m(t), Y_s(t), I(t)) + z(u_m(t) + u_s(t)) - r\, L_m(t)\epsilon_m - r L_s(t)\epsilon_s\Big\} e^{-rt}\, dt, \quad (53)$$

subject to, for each sector $i \in \{s, m\}$,

$$\dot{n}_i(t) = m_i(t) - s_i n_i(t), \quad (54)$$

$$\dot{Y}_i(t) = -s_i Y_i(t) + m_i(t) f_i(k_i(t)), \quad (55)$$

$$u_i(t) = L_i(t) - n_i(t), \quad (56)$$

$$m_i(t) = q_i\big(\theta_i(t)\big)\, \theta_i(t)\, \big(L_i(t) - n_i(t)\big), \quad (57)$$

$$v_i(t) = \theta_i(t)\, \big(L_i(t) - n_i(t)\big), \quad (58)$$

$$L_m(t) = L - L_s(t). \quad (59)$$

[8]Note that skill acquisition cost ϵ_i in the model is incurred once when workers choose a sector. In the planner's objective function, we can represent this one-time cost in an equivalent flow term $r\epsilon_i$ because $\int_0^\infty re^{-rt}dt = 1$.

Proposition 3 (Social Planner). *In steady state, the social planner's choices of k_i, θ_i and L_s are characterized by*

$$\{\textit{Capital choice}\}: \quad P_k = \frac{p_i f'(k_i)}{r+s_i} \frac{q_i(\theta_i)}{r+s_i+q_i(\theta_i)}, \quad i \in \{m,s\} \tag{60}$$

$$\{\textit{Tightness}\}: \quad \frac{P_k\, k_i}{P} = \frac{q_i(\theta_i)\,(1-\eta)\,(p_i f(k_i)/P - z)}{(r+s_i)D_i(\eta)}, \quad i \in \{m,s\} \tag{61}$$

$$\{\textit{Sectoral choice}\}: \quad r(\epsilon_m - \epsilon_s) = \theta_m q_m(\theta_m)\,\eta\Phi_m - \theta_s q_s(\theta_s)\,\eta\Phi_s, \tag{62}$$

$$\textit{where} \quad \Phi_i = \frac{p_i f_i(k_i)/P - z}{D_i(\eta)}, \tag{63}$$

$$D_i(\eta) = r + s_i + (1-\eta)q_i(\theta_i) + \eta\theta_i q_i(\theta_i). \tag{64}$$

$$\eta \equiv \frac{\partial log M(u,v)}{\partial log(u)}. \tag{65}$$

Remarks. Equation (60) characterizes the social planner's optimal choice of k_i. The right-hand side is the marginal social return to capital: $p_i f_i'(k_i)$, the marginal product of capital, discounted by the interest rate and separation rate, and adjusted by the probability that the vacancy is filled, $\frac{q_i}{r+s_i+q_i}$. Equation (61) and (62) are the planner's counterparts to the decentralized equilibrium conditions (51) and (52), except that β is replaced with η, the elasticity of matches with respect to employment.

The planner internalizes how changes in labor market tightness affect match formation, so the relevant object is the elasticity η. In the decentralized equilibrium, however, the corresponding condition depends on workers' bargaining power β, which governs how surplus is privately shared. The Hosios condition, $\beta = \eta$, is therefore a natural benchmark: when it holds, the decentralized equilibrium coincides with the planner on search externality, but the capital holdup distortion remains.

5 Efficiency

To analyze the sources of inefficiency, we compare the optimality conditions of decentralized equilibrium (DE) with those of social planner's allocation (SP). Let $\{k_i^{DE}, \theta_i^{DE}, L_i^{DE}\}$ denote the allocation that satisfies DE's conditions (50)-(52) and $\{k_i^{SP}, \theta_i^{SP}, L_i^{SP}\}$ as the bundle satisfying SP's conditions (60)-(62).

5.1 Hosios condition $\beta = \eta$.

Proposition 4. *Under the Hosios condition, $\beta = \eta$,*

(i) The decentralized equilibrium is constrained inefficient;

(ii) If $k_i = k_i^{SP}$ is fixed exogenously, then $\theta_i^{DE} = \theta_i^{SP}$ and $L_i^{DE} = L_i^{SP}$.

Remarks. Proposition 4 (i) follows directly from comparing the private and social marginal return to capital in (50) and (60). Define the capital wedge Ω_i^K as the ratio between the private and social marginal return to capital:

$$\Omega_i^K \equiv \frac{\text{Marginal return to } k_i^{DE}}{\text{Marginal return to } k_i^{SP}}$$
$$\equiv \frac{\frac{q_i(\theta_i)(1-\beta)}{r+s_i}\frac{p_i f'(k_i)}{r+s_i+(1-\beta)q_i(\theta_i)}}{\frac{p_i f'(k_i)}{r+s_i}\frac{q_i(\theta_i)}{r+s_i+q_i(\theta_i)}} = (1-\beta)\frac{r+s_i+q_i(\theta_i)}{r+s_i+(1-\beta)\,q_i(\theta_i)} < 1. \tag{66}$$

Therefore, the private marginal return to capital in DE is smaller than the social marginal return to capital in SP. However, when $\beta = \eta$, (51)-(52) in DE are identical to (61)-(62), which leads to Proposition 4 (ii).

Proposition 5. *Under Hosios condition $\beta = \eta$, a deficit-neutral policy consisting of a one-time investment subsidy τ_i^{K*} per unit of investment and a one-off lump-sum tax on firms τ_i^{T*} per vacancy implements the social planner's allocation:*

$$\tau_i^{K*} = 1 - \Omega_i^K = 1 - \frac{(1-\beta)(r+s_i+q_i(\theta_i^{SP}))}{r+s_i+(1-\beta)q_i(\theta_i^{SP})} > 0$$
$$\tau_i^{T*} = -\tau^{K*}P_k k_i^{SP}/P < 0$$

The proof of Proposition 5 is available in Appendix A. Intuitively, the investment subsidy τ_i^{K*} corrects the inefficiency arising from capital hold-up and induces firms to choose higher k^i. This subsidy also raises firms' profits, encouraging (inefficiently) more firms to enter. Thus, a lump-sum tax on firm entry, τ_i^T, is needed to prevent inefficient over-entry. Another key implication is that once the capital level is efficient, there is no need to subsidize workers' training as long as Hosios condition holds, as higher per worker capital endogenously raises workers' incentives to enter the manufacturing sector.

What are the real-world counterparts of τ_i^K and τ_i^T? The investment subsidy τ_i^K maps naturally into policies that reduce the net cost of investment, such as accelerated depreciation allowances, investment tax credits, or direct investment subsidies. The counterpart of the lump-sum tax τ_i^T is more subtle. It is best interpreted as a firm contribution that is proportional to employment but does not vary one-to-one with the total wage bill. Examples include a flat payroll tax, a flat social security contribution, or payroll taxes that apply only up to a low wage threshold.

5.2 Empirical evidence: skill supply responds to skill demand.

A key premise of Proposition 5 is that workers' educational choices respond to shifts in skill demand. To illustrate this mechanism empirically, we follow Han and Winters (2020) to use the response of petroleum-engineering enrollment to movements in real oil prices as evidence that skill supply adjusts to demand. This relationship has several advantages. First, the real oil price can plausibly be treated as an exogenous shock to labor demand in the oil industry. When oil prices rise, an increase in petroleum-engineering enrollment can be interpreted as a causal response to this demand shock. Second, unlike broad fields such as mathematics, petroleum engineering is narrowly defined and closely tied to the oil industry. This tighter mapping helps isolate the demand channel and reduces concerns that the observed enrollment response is driven by other contemporaneous shocks.

Our real oil price series is constructed from annual WTI price deflated by the U.S. CPI. Both series are obtained from FRED. There have been two real oil price booms since 1960: one beginning in 1973 during the energy crisis, and another starting in the mid-2000s.

We use the 2010-24 American Community Survey (ACS) to construct a cohort-level measure of the petroleum-engineering enrollment shares going back to 1960. The ACS is an annual national survey of the U.S. population. Relevant to our purpose, the survey asks respondents' demographic and socioeconomic characteristics, such as age, gender, income level, birth place, education level, and college major when applicable, along with sample weights. To construct the share of petroleum-engineering enrollment share by year, we follow Han and Winters (2020) and extend their sample through the late 2010s to include the oil price boom of the 2000s. For each cohort year t between 1960 and 2019, we pool the 2010-24 ACS microdata and identify respondents who turned 18 years old in year t. We restrict the sample to individuals born in the largest three oil-producing states (Texas, North Dakota, and New Mexico), under the premise that they are more likely to observe oil industry labor demand conditions, and thus adjust their field-of-study choices when oil prices rise. We then compute the weighted number of respondents in cohort t who reported major is petroleum engineering. Dividing by the weighted sum of enrollment in the same cohort yields the petroleum-engineering share.

Figure 2 plots the real oil price (right axis, dashed line) together with the petroleum-engineering enrollment share for cohorts born in Texas, North Dakota, and New Mexico (left axis, solid line). A clear co-movement emerges: petroleum-engineering enrollment rose during the two major oil price booms in the 1970s and mid-2000s, and declined when oil prices fell. This pattern was more pronounced during the first boom, but the response remains evident in the 2000s episode. Overall, the figure provides suggestive evidence that educational choices respond to changes in industry labor demand.

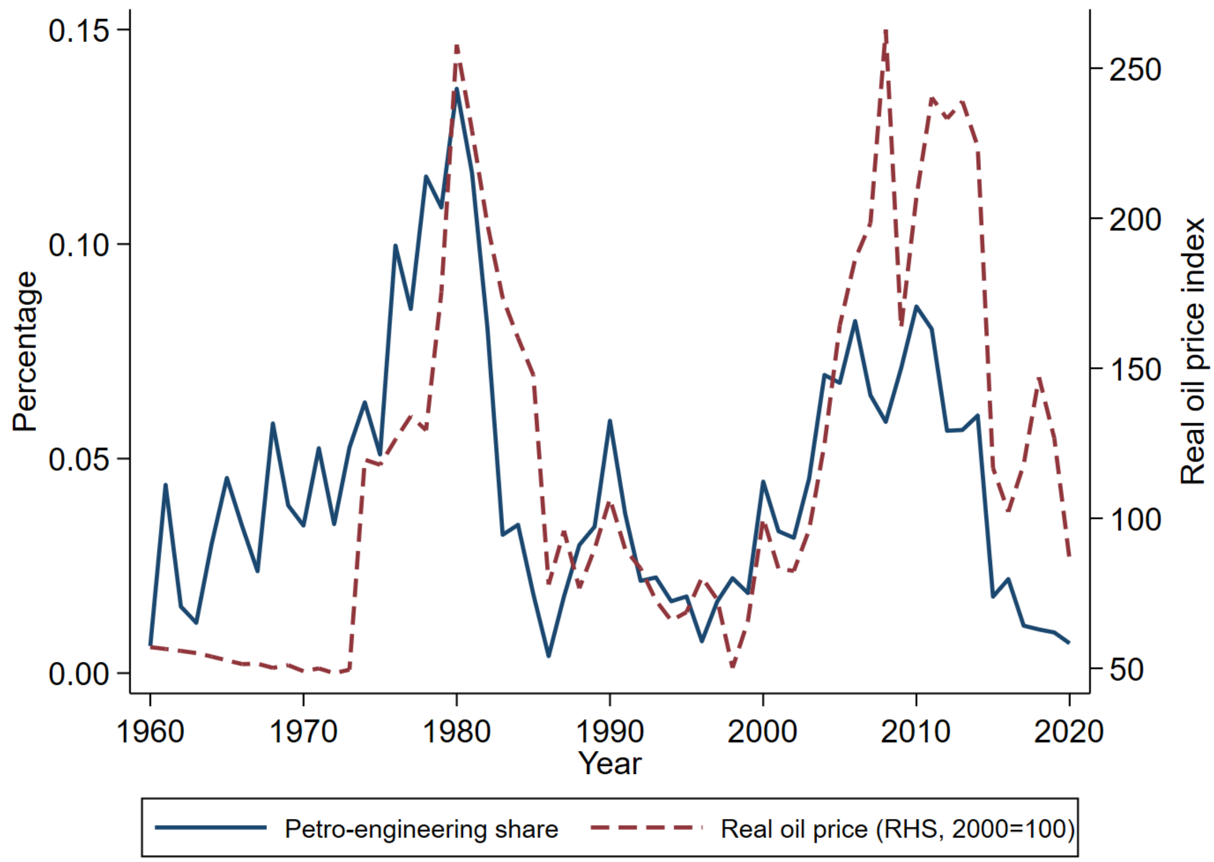


Figure 2. **Real oil prices and petroleum engineering enrollment share.** The figure plots the petroleum engineering major share among cohorts born in Texas, North Dakota, and New Mexico (left axis) against the annual real oil price (2000=100, right axis). Cohorts are indexed by the year individuals turned age 18.

5.3 Inefficiency when Hosios condition fails

5.3.1 The empirically relevant case: $\eta > \beta$

The preceding subsection focused on the Hosios benchmark $\beta = \eta$, under which the decentralized equilibrium internalizes the standard search externality. Empirically, however, a relevant case is that workers' bargaining power β is lower than the matching elasticity η, that is, $\eta > \beta$.

The matching elasticity with respect to unemployment, η, is relatively well identified. Petrongolo and Pissarides (2001) survey the empirical literature and report estimates typically in the range of 0.5 to 0.7. More recent studies, such as (Borowczyk-Martins et al., 2013), are broadly consistent with this range. In contrast, estimates of workers' bargaining power, β, are more dispersed but generally suggest values below η. Hagedorn and Manovskii (2008) estimate $\beta \approx 0.05$ in their baseline specification, though this rises to about 0.46 when capital is incorporated into the model. Flinn (2006) estimates bargaining power in the range of 0.3-0.4. More recent evidence points to a secular decline in workers' bargaining power: Azar et al. (2022) document falling labor shares and attribute this partly to declining worker bargaining power, consistent with lower unionization and greater labor market concentration.

Proposition 6. *When $\eta > \beta$ and $\epsilon_m > \epsilon_s$, the decentralized equilibrium is inefficient even if the capital is fixed exogenously at the social planner's level, $k_i = k_i^{SP}$.*

The gap between the tightness conditions in the decentralized equilibrium, equation(51), and in the social planner's allocation, equation (61), reflects the familiar inefficiency that arises when Hosios condition fails.

What is new in our two-sector environment is an additional wedge in workers' sectoral choices. Substitute the planner's tightness condition (61) into (62) and the decentralized equilibrium's tightness condition (51) into (52), we have

$$r(\epsilon_m - \epsilon_s) = \frac{\eta}{1-\eta}[\theta_m P_k k_m (r + s_m) - \theta_s P_k k_s (r + s_s)], \tag{67}$$

$$r(\epsilon_m - \epsilon_s) = \frac{\beta}{1-\beta}[\theta_m P_k k_m (r + s_m) - \theta_s P_k k_s (r + s_s)]. \tag{68}$$

Even holding θ_i and k_i fixed, it is immediate that when $\eta > \beta$ and $\epsilon_m > \epsilon_s$, the right-hand side of (67) exceeds that of (68). Intuitively, the right hand side of (68) represents the flow surplus gained by an unemployed worker who enters manufacturing rather than services. The term $P_k k_i (r + s_i)$ is the user cost of firm investment, and multiplying by θ_i

scales this cost by the number of vacancies per unemployed worker. Finally, the factor $\frac{\beta}{1-\beta}$ converts the firm's flow surplus into worker's surplus via Nash bargaining.

Because $\epsilon_m > \epsilon_s$, a worker must obtain more surplus in manufacturing than in services to compensate for the higher educational cost. When $\beta < \eta$, the decentralized equilibrium understates the social return to shifting workers toward manufacturing.

Proposition 7. *Under $\beta < \eta$, the social planner's allocation can be implemented using three policy instruments $\{\tau_i^{K**}, \tau_i^{T**}, \tau_i^{L**}\}$: a sector-specific investment subsidy, a sector-specific lump-sum tax on firms, and a training subsidy for manufacturing workers. Specifically,*

$$\tau_i^{K**} = 1 - \frac{(1-\beta)(r+s_i+q_i)}{r+s_i+(1-\beta)q_i} > 0, \tag{69}$$

$$\tau_i^{T**} = \frac{q_i(1-\eta)}{(r+s_i)}\Phi_i - \tau_i^{K**}P_k k_i/P - \frac{q_i(1-\beta)[p_i f_i(k_i)/P - z]}{(r+s_i)D_i(\beta)}, \tag{70}$$

$$\tau_m^{L**} = r(\epsilon_m - \epsilon_s) - [\theta_m q_m \,\eta\, \Phi_m - \theta_s q_s \,\eta\, \Phi_s] \tag{71}$$

$$- [\theta_m q_m \beta \frac{p_m f_m(k_m)/P - z}{D_m(\beta)} - \theta_s q_s \beta \frac{p_s f_s(k_s)/P - z}{D_s(\beta)}], \tag{72}$$

$$\tau_s^{L**} = 0, \tag{73}$$

where $\{\theta_i, k_i, q_i, L_i\} = \{\theta_i^{SP}, k_i^{SP}, q_i^{SP}, L_i^{SP}\}$, and

$$D_i(\beta) = r + s_i + (1-\beta)q_i + \beta\theta_i q_i,$$
$$\Phi_i = \frac{p_i f_i(k_i)/P - z}{D_i(\eta)}.$$

6 Calibration

We calibrate the benchmark model at an annual frequency to Chile. Several parameters are set exogenously based on direct empirical counterparts or standard values in the literature. While we target Chilean moments whenever possible, for some parameters we rely on U.S. evidence because of data limitations.

We set the elasticity of substitution between domestic and foreign services $\nu = 2$, within the range commonly used in applied trade models, for example, in Dimaranan et al. (2006). This implies that domestic and foreign services are gross substitutes. We normalize the foreign prices p_m and p_s^F to one. We set the total labor force to one and choose the sum of manufacturing and service labor forces $L = 0.94$, so that the implied agricultural labor force – treated as exogenous – is 6 percent of the total labor force. This targets Chile's observed agricultural employment share, which is about 6% in 2024. We assume the production functions are in Cobb-Douglas form: $f_i(k_i) = A_i k_i^{\xi_i}$. We normalize the service sector productivity A_s to one. We set ξ_m and ξ_s, to be 0.65 and 0.3, respectively. These values are consistent with the evidence that capital's income share is substantially higher in industry than in services (0.63 vs. 0.34 in 2022, based on Comisión Nacional de Productividad (CNEP) data). Because sectoral factor-income shares are measured with error, we treat these numbers as informative benchmarks rather than exact targets.

For labor market parameters, we normalize the matching efficiency to one, following standard practice (Mortensen and Pissarides 1994; Petrongolo and Pissarides 2001). We set the elasticity of matches with respect to unemployment to $\eta = 0.5$, the midpoint of the empirical estimates surveyed by Petrongolo and Pissarides (2001), and in line with standard calibrations (e.g., Hall and Milgrom 2008; Menzio et al. 2016). In the benchmark calibration, we set workers' bargaining power to $\beta = \eta$, so that the standard matching externality is shut down. We will report the results for the case $\beta < \eta$.

Table 2: Exogenously Set Parameters:

Parameter	Description	Value	Source
ν	CES aggregator coefficient	2	Dimaranan et al. (2006)
m	Matching efficiency	1	Normalization
η	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
β	Worker bargaining weight	0.5	Hosios condition
A_s	Services technology	1	Normalization
p_m	m goods price	1	Normalization
p_s^F	Foreign service goods price	1	Normalization
L	sum of L_m and L_s	0.94	Agriculture labor force $\sim$ 0.06
ζ_m	Capital share in manufacturing	0.65	CNEP
ζ_s	Capital share in services	0.3	CNEP

Endogenously calibrated parameters. The remaining parameters are calibrated to match moments in the decentralized equilibrium. Their values are reported in Table 3. We choose the preference weight on manufacturing goods, α, so that $L_m = 21.7\%$ in equilibrium, matching Chile's industrial employment share in 2024. The separation rates, s_m and s_s, are pinned down by sectoral unemployment rates. Specifically, we target a 6.9% unemployment rate in each sector, which equals the average unemployment rate in Chile during 2015-19.[9] The time discount rate r is pinned down by the aggregate capital-to-output ratio $\frac{P_k K}{Y^N}$. In the model, total capital is $K \equiv \sum_i k_i(n_i + v_i)$, and nominal output

[9] Arroyo et al. (2024) documents that unemployment rates for workers who chose STEM majors are similar to those in non-STEM tracks.

$Y^N \equiv p_m Y_m + p_s Y_s$. We set r to 0.06, which implies a capital-to-output ratio $K/Y^N = 2.9$. This matches Chile's non-mining, non-agriculture capital-to-output ratio in 2022 (2.9),[10] and is also consistent with the range of estimates in Feenstra et al. (2015) for a broader set of countries.[11]

We calibrate the relative productivity, A_m/A_s, to match the sectoral wage ratio w_m/w_s, which Elvery and Dunn (2021) estimate to be 4-14 percent for the United States. Finally, we calibrate the educational cost differential, $\epsilon_m - \epsilon_s$, to 70 percent of w_m based on Altonji and Zimmerman (2018), who document training cost by major. Using their estimates, we compare the average training costs in STEM and non-STEM fields, and obtain a difference equivalent to 70-75 percent of the per worker income. The parameter χ governs the equilibrium service import-to-GDP ratio. Because of the balanced trade assumption, the closest data counterpart is goods trade surplus-to-GDP ratio. We choose $1-\chi = 0.0044$ so that the goods trade surplus is about 5.5 percent of GDP , matching Chile's average trade surplus in 2023-24. Finally, we set unemployment income to $z = 2.5$ which implies a home production-to-wage ratio of 0.30 in equilibrium, within the range of 0.25-0.35 estimated by House et al. (2008).

Table 3: Internally Calibrated Parameters.

Parameter	**Description**	**Value**	Data Moment
α	Preference weight on m goods	0.174	Industrial employment share
s_m	Separation rate in m	0.048	Unemployment rate in m
s_s	Separation rate in s	0.115	Unemployment rate in s
r	Discount rate (annual)	0.06	Capital-output ratio
$1-\chi$	Service import weight	0.0044	Goods trade balance-to-GDP
A_m/A_s	Relative productivity	2.0	Sectoral wage ratio
$\epsilon_m - \epsilon_s$	Education cost differential	6.0	Δ training cost-to-labor income
z	Home production utility	2.5	Home production-to-wage

[10]Chile's mining sector is capital intensive and accounts for about 10–12 percent of aggregate output. We therefore target the capital-to-output ratio excluding mining to be more representative of other small open economies. Including mining, Chile's aggregate capital-to-output ratio is about 3.3 in 2022.

[11]See Appendix C in Feenstra et al. (2015).

Table 4: Matching the calibration targets and non-targeted moments.

Moment	Target	Model
Share of industrial labor, L_m	21.7%	21.7%
Unemployment rate (m)	6.9%	6.9%
Unemployment rate (s)	6.9%	6.9%
Capital-to-output ratio	2.9	2.9
Goods trade surplus-to-GDP	5.4%	5.4%
Wage ratio w_m/w_s	1.09	1.09
Training cost differential-to-wage	0.72	0.71
Home production-to-wage	0.30	0.29

7 Quantitative Results

7.1 Decentralized equilibrium versus social planner.

Table 5 compares the decentralized equilibrium (DE) with the social planner's allocation (SP). Relative to the decentralized equilibrium, the planner chooses a higher capital–output ratio, $K/Y = 3.3$ instead of 2.9. This reflects the holdup inefficiency in the decentralized economy: because wages are bargained after capital is sunk, firms internalize only a fraction of the marginal match surplus when choosing vacancy capital, which leads to underinvestment.

Capital deepening raises output per worker in both sectors, with a larger increase in manufacturing because production there is more capital intensive. Relative to the decentralized equilibrium, real labor productivity, $f_m(k_m)$, is 13.7 percent higher, compared with 10.0 percent in services. Total output also rises more in manufacturing than in services, by 17.9 percent versus 7.4 percent.

The social planner also chooses a higher industrial labor force share than the DE: L_m rises by 1.0 percentage point. Intuitively, because manufacturing is more capital intensive, correcting the holdup wedge raises match surplus in manufacturing more than in services. This increases the marginal gain from reallocating one more worker from services to manufacturing. The open-economy goods block dampens the relative-price adjustment, so the reallocation occurs primarily through labor quantities rather than relative prices.

Unemployment rates are **higher under the social planner** in both sectors. Intuitively, the planner chooses higher capital per worker, k_i, than in the decentralized equilibrium. Because vacancy creation requires installing job-specific capital, a higher k_i raises the cost of vacancy creation. The planner hence chooses lower market tightness, $\theta_i^{SP} < \theta_i^{DE}$, which reduces job-finding rates and increases unemployment rates. Finally, real output is about 11.3% higher in the planner's allocation, driven almost entirely by capital deepening and the resulting increase in labor productivity.

Table 5: DE and SP Allocation under Endogenous Capital Level

Moments	**DE**	**SP**	Δ
Labor force share (m), L_m	21.7%	22.7%	+1.0 pp
Employment share (m), $n_m/(n+L_a)$	21.6%	22.7%	+1.1 pp
Unemployment rate (m), u_m/L_m	6.9%	7.5%	+0.6 pp
Unemployment rate (s), u_s/L_s	6.9%	7.9%	+1.0 pp
Tightness, θ_m	0.42	0.35	-0.07
Tightness, θ_s	2.40	1.77	-0.63
Capital per worker (m), k_m	841	1024	$+21.8\%$
Capital per worker (s), k_s	84	116	$+37.5\%$
Capital-to-output ratio, K/Y	2.9	3.3	$+0.4$
Total capital, K	242	309	$+27.5\%$
Relative prices, p_s/p_m	20.1	20.1	-0.1%
Real labor productivity, $f_m(k_m)$	79.6	90.5	$+13.7\%$
Real labor productivity, $f_s(k_s)$	3.8	4.2	$+10.0\%$
Output (m), Y_m	32.2	38.0	$+17.9\%$
Output (s), Y_s	2.5	2.7	$+7.4\%$
Real output, Y	11.8	13.1	$+11.5\%$

7.2 Policy when $\beta = \eta$

7.2.1 Policies that implement the planner's allocation.

Under $\beta = \eta$, the decentralized equilibrium is inefficient only because vacancy capital is sunk before bargaining, which generates the holdup wedge. Proposition 5 shows that the planner's allocation can be implemented by a subsidy τ_i^{K*} on newly created vacancies, together with an offsetting lump sum tax τ^{T*_i} that preserves the free entry condition. Under the baseline calibration, this yields $\tau_m^{K*} = 5.6\%$ and $\tau_s^{K*} = 15.7\%$. Because the subsidy τ_i^{K*} applies to the flow of newly created vacancies, the aggregate fiscal cost of the investment subsidy measured in final consumption, is

$$\text{Sub}_K = \sum_i \tau_i^{K*} \frac{P_k\, k_i\, x_i}{P} = \sum_i \tau_i^{K*} \frac{P_k\, k_i\, s_i\,(n_i + v_i)}{P}. \tag{74}$$

where the second equality uses the steady-state condition $x_i = s_i(n_i + v_i)$. Under the baseline calibration, this subsidy is 2.4% of GDP per year. It is fully financed by the lump sum tax τ^{T*}.

7.2.2 Investment subsidy only.

In practice, the government may not be able to levy employment-based lump sum taxes due to political constraints, and sector-specific investment subsidies may be difficult to administer. Table 6 therefore reports equilibrium outcomes under three policy configurations. DE1 implements the planner's allocation. DE2 sets $\tau^T = 0$ while keeping sector-specific investment subsidies. DE3 imposes a sector-uniform subsidy and again sets $\tau_i^T = 0$.

Labor market outcomes. Removing the lump-sum tax in DE2 and DE3 induces over-entry. Market tightness, θ_i, rises in both sectors, and vacancy-filling rates, q_i, fall. The higher tightness increases job-finding rates and lowers unemployment. The industrial labor force share rises further in DE2 relative to DE1, reflecting stronger vacancy creation in manufacturing when the lump sum tax is absent.

Capital, output, and welfare. Relative to DE1, DE2 and DE3 feature slightly lower capital-to-output ratios. Intuitively, when the lump-sum tax is absent, firms have stronger incentives to create vacancies than to deepen capital. Output differences are modest: Y_m is slightly higher in DE2 while Y_s is slightly lower. Welfare in DE2 is slightly lower than that in DE1 because the lower unemployment also means less home production.

Fiscal cost. DE1 is fiscally neutral, whereas DE2 and DE3 require net fiscal costs of 2.4 and 2.1 percent of GDP, respectively.

Table 6: Equilibrium Allocations under Taxes and Subsidies when $\beta = \eta$

Moment	**DE1**	**DE2**	**DE3**
Description	Implements SP	Only τ^K	Only uniform τ^K
τ_i^K	$\tau_i^K = \tau_i^{K*}$	$\tau_i^K = \tau_i^{K*}$	$\tau_i^K = \tau_m^{K*}$
τ_i^T	$\tau_i^T = \tau_i^{T*}$	0	0
Industrial labor force, L_m	22.7%	23.2%	22.7%
Industrial unemploy. rate, u_m/L_m	7.5%	6.8%	6.8%
Services unemploy. rate, u_s/L_s	7.9%	6.8%	6.8%
Capital-to-output ratio, K/Y	3.3	3.3	3.1
Tightness, θ_m	0.35	0.44	0.43
Tightness, θ_s	1.77	2.52	2.49
Real output, Y_m	38.0	38.3	37.4
Real output, Y_s	2.7	2.7	2.64
Welfare, W	4.18	4.15	4.09
Annual fiscal cost (% of GDP)	0	2.4	2.1

7.3 Optimal taxes and subsidies when $\beta < \eta$.

We now relax the Hosios condition and consider the empirically relevant case in which workers' bargaining power, β, is lower than the matching elasticity, η. We set $\beta = 0.3$ while holding all other parameters, including $\eta = 0.5$, at their benchmark values. This departure introduces the standard search externality, so that vacancy creation is no longer efficient. It also generates the additional wedge in workers' sectoral choices that is central to our model: because workers capture too small a share of match surplus, their sectoral choices remain distorted even after the vacancy-creation externality is corrected.

To illustrate this point, Table 7 reports the equilibrium outcomes under three policy settings. First, DE-A is the decentralized equilibrium under $(\beta, \eta) = (0.3, 0.5)$ with no policy instruments. Second, DE-B implements the social planner's allocation using the full policy package: sector-specific investment subsidies and lump sum taxes, $\{\tau_i^{K**}, \tau_i^{T**}\}_{i \in \{m,s\}}$, together with a manufacturing training subsidy τ_m^{L**}.[12] Third, DE-C applies the same firm-side policies but sets τ_m^L =0. This last case isolates the additional sectoral choice wedge: if correcting firm-side distortions were sufficient, DE-C would coincide with the planner's allocation.

Relative to the benchmark decentralized equilibrium with $\beta = \eta = 0.5$ (column DE), lowering workers' bargaining power to $\beta = 0.3$ in DE-A raises firms' incentives to create vacancies, resulting in substantially higher market tightness and lower unemployment in both sectors. Manufacturing output increases modestly, while service output changes little. Despite these gains, welfare falls slightly as the labor market becomes excessively tight, allocating too many resources to vacancy creation and reducing the time spent on home production.

The comparison between DE-B and DE-C highlights the additional wedge in sectoral choice. By construction, DE-B coincides with the planner's allocation. In contrast, when the training subsidy is shut down, the labor force share in manufacturing falls sharply in DE-C relative to DE-B. The mechanism is precisely the sectoral wedge implied by $\beta < \eta$: even after firm-side distortions are corrected, workers capture too small a share of surplus to justify the move into manufacturing. A targeted subsidy to manufacturing skill acquisition therefore plays a distinct role in correcting this wedge.

[12] Note that $\tau_s^{L**} = 0$ in Proposition 7.

Table 7: Equilibrium allocations, $\beta < \eta$

Moment	DE	DE-A	DE-B	DE-C
Description	Benchmark	DE ($\beta < \eta$)	Implements SP	No training subsidy
β	0.5	0.3	0.3	0.3
η	0.5	0.5	0.5	0.5
τ_i^K	0	0	$\tau_i^K = \tau_i^{K**}$	$\tau_i^K = \tau_i^{K**}$
τ_i^T	0	0	$\tau_i^T = \tau_i^{T**}$	$\tau_i^T = \tau_i^{T**}$
τ_m^L	0	0	$\tau_m^L = \tau_m^{L**}$	0
Industry labor force, L_m	21.7%	21.9%	22.7%	20.3%
Industry unemp. rate, u_m/L_m	6.9%	4.6%	7.5%	7.1%
Services unemp. rate, u_s/L_s	6.9%	4.8%	7.9%	8.4%
Capital-to-output ratio, K/Y	2.9	3.0	3.3	3.3
Tightness, θ_m	0.42	0.98	0.35	0.40
Tightness, θ_s	2.40	5.76	1.77	1.59
Real output, Y_m	32.2	32.7	38.0	33.8
Real output, Y_s	2.5	2.6	2.7	2.7
Real consumption C	3.8	3.8	4.1	3.9
Welfare, W	3.9	3.8	4.2	4.0

7.4 Trade openness and sectoral employment

In our model, greater service trade openness, captured by a higher value of $1-\chi$, raises the demand for imported services. Under balanced trade, higher service imports require higher manufacturing exports, which raises labor demand in manufacturing and shifts workers toward that sector. This mechanism is present even in a frictionless small open economy.

Search frictions, however, affect the adjustment margin. As service trade openness rises, imported services account for a larger share of the service consumption basket. Because the imported-service price is lower than domestically produced services, the service price index P_s falls. This lowers the relative prices of final consumption compared with both manufacturing and services, P/p_m and P/p_s, and reduces the value of home production z relative to wages in both sectors and thereby lowers equilibrium unemployment.

To illustrate the impact of greater service trade openness, we increase the service-import weight in the service basket from $1-\chi$ to $1-\chi'$, where $1-\chi' = 3(1-\chi)$. Several changes to the equilibrium follow. First, the service import-to-GDP ratio increases from 5.4 percent to 14.0 percent. Second, because trade is balanced, manufacturing exports also rise, raising manufacturing labor force share from 22.7 percent to 28.9 percent. Third, unemployment rates in both sectors decline, the aggregate unemployment rate declines from 6.9 percent to 6.7 percent. This decline is largely driven by the decline of service price P_s and final good consumption price P relative to p_m, which reduces the value of home production relative to wages.

8 Conclusion

This paper studies the chicken-and-egg problem in manufacturing development: should policy prioritize boosting investment or closing sector-specific skill gaps? We develop a two-sector small open economy model with DMP-style search and matching frictions to address this question. Our key mechanism is job-specific capital chosen before wage bargaining, which creates a capital holdup distortion. The policy implication depends on the Hosios condition. When the Hosios condition holds, correcting capital underinvestment alone is sufficient to replicate the planner's allocation. The optimal policy consists of an investment subsidy, fully financed by a lump sum tax on firms. When the Hosios condition fails, an additional wedge distorts workers' sectoral choices beyond the standard search externality, implying a role for targeted training subsidies alongside investment incentives.

Appendix A. Decentralized equilibrium with policy instruments

This appendix derives the decentralized equilibrium (DE) with policy instruments: τ_i^K for investment subsidy, τ^L for education subsidy, and τ^T for lump-sum transfers. Starting from the Bellman equations:

$$rJ_i^U = \theta_i q_i(\theta_i)\big(J_i^E(k_i^*) - J_i^U\big) + \tau_i^L + z, \tag{A.1}$$

$$rJ_i^E(k_i) = w_i(k_i)/P + s_i\big(J_i^U - J_i^E(k_i)\big) + \tau_i^L, \tag{A.2}$$

$$rJ_i^V(k_i) = q_i(\theta_i)\big(J_i^F(k_i) - J_i^V(k_i)\big) - s_i J_i^V(k_i), \tag{A.3}$$

$$rJ_i^F(k_i) = p_i f(k_i)/P - w_i(k_i)/P - s_i J_i^F(k_i). \tag{A.4}$$

$$k_i^* = argmax_{k_i} J_i^V(k_i) \tag{A.5}$$

The equilibrium free-entry condition implies

$$\tau_i^T + J_i^V(k_i) = (1 - \tau_i^K)P_k k_i/P \tag{A.6}$$

For each vacancy, it is subsidized with a one-off investment credit $\tau_i^K P_k k_i$. When it opens the vacancy, it also receives a lump-sum transfer τ_i^T[13]. Each worker who chooses into sector i receives a subsidy flow τ_i. The total subsidy cost is financed by a lump sum tax T. The government's net fiscal transfer is:

$$T = \sum_i \tau_i^K P_k k_i s_i(n_i + v_i)/P + \sum_i \tau_i^T s_i(n_i + v_i) + \sum_i \tau_i^L L_i \tag{A.7}$$

From (A.3) and (A.4), we have

$$J_i^F(k_i) = \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i)} \tag{A.8}$$

$$\begin{aligned} J_i^V(k_i) &= \frac{q_i(\theta_i)J_i^F(k_i)}{r + s_i + q_i(\theta_i)} \\ &= \frac{q_i(\theta_i)}{r + s_i + q_i(\theta_i)} \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i)} \end{aligned} \tag{A.9}$$

$$J_i^F(k_i) - J_i^V(k_i) = \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i + q_i(\theta_i))} \tag{A.10}$$

[13]When $\tau_i^T < 0$, this is a lump-sum tax on firms.

From (A.9) and the standard first order condition $J_i^{V\prime}(k_i) = (1-\tau_i^K)\frac{P_k}{P}$, we have

$$(1-\tau_i^K)P_k = \frac{q_i(\theta_i)}{r+s_i+q_i(\theta_i)} \frac{p_i f'(k_i) - w_i'(k_i)}{r+s_i}. \tag{A.11}$$

Next, we solve for $p_i f(k_i) - w_i(k_i)$, taking J_i^U as given. Subtracting (A.3) from (A.4), we have

$$\begin{aligned}[p_i f(k_i) - w_i(k_i)]/P &= \big(r+s_i+q_i(\theta_i)\big)\big(J_i^F(k_i) - J_i^V(k_i)\big) \\ &= \big(r+s_i+q_i(\theta_i)\big)(1-\beta_i)S_i(k_i)\end{aligned} \tag{A.12}$$

From (A.2), we have

$$\begin{aligned}w_i(k_i)/P - rJ_i^U + \tau_i^L &= (r+s_i)\big(J_i^E(k_i) - J_i^U\big) \\ &= (r+s_i)\beta_i S_i(k_i)\end{aligned} \tag{A.13}$$

Adding (A.12) and (A.13), we have

$$p_i f(k_i)/P - rJ_i^U + \tau_i^L = \big[r+s_i+(1-\beta_i)q_i(\theta_i)\big]S_i(k_i), \tag{A.14}$$

Hence the total surplus from a match with capital k_i is

$$S_i(k_i) = \frac{p_i f(k_i)/P - rJ_i^U + \tau_i^L}{r+s_i+(1-\beta_i)q_i(\theta_i)}, \tag{A.15}$$

Substitute (A.14) and (A.15) into (A.12),

$$[p_i f(k_i) - w_i(k_i)]/P = \frac{(1-\beta_i)\big(r+s_i+q_i(\theta_i)\big)}{r+s_i+(1-\beta_i)q_i(\theta_i)}\left[p_i f(k_i)/P - rJ_i^U + \tau_i^L\right] \tag{A.16}$$

Take the derivative with to k_i, we have the optimality condition for k_i with policy instruments:

$$(1-\tau_i^K)P_k = \frac{q_i(\theta_i)(1-\beta_i)}{r+s_i} \frac{p_i f'(k_i)}{r+s_i+(1-\beta_i)q_i(\theta_i)} \tag{A.17}$$

Next we derive the free-entry condition. From the Bellman equations, we easily have

$$r\big(J_i^E(k_i) - J_i^U\big) = w_i(k_i)/P - z - \big(s_i+\theta_i q_i(\theta_i)\big)\big(J_i^E(k_i) - J_i^U\big), \tag{A.18}$$

$$r\big(J_i^F(k_i) - J_i^V(k_i)\big) = [p_i f(k_i) - w_i(k_i)]/P - \big(s_i+q_i(\theta_i)\big)\big(J_i^F(k_i) - J_i^V(k_i)\big). \tag{A.19}$$

Substitute the Nash bargaining outcome $J_i^E(k_i) - J_i^U = \beta_i S_i(k_i)$ and $J_i^F(k_i) - J_i^V(k_i) =$

$(1-\beta_i)S_i(k_i)$, we have

$$S_i(k_i) = [p_i f(k_i)/P - z]/D_i \tag{A.20}$$

where $D_i = r + s_i + \beta_i\theta_i q_i(\theta_i) + (1-\beta_i)q_i(\theta_i)$.

From (A.3), we have at the steady state,

$$\begin{aligned} J_i^V(k_i) &= \frac{q_i(\theta_i)}{r+s_i}\big(J_i^F(k_i) - J_i^V(k_i)\big) \\ &= \frac{q_i(\theta_i)}{r+s_i}(1-\beta_i)S_i(k_i) \\ &= q_i(\theta_i)(1-\beta_i)\frac{p_i f(k_i)/P - z}{D_i\,(r+s_i)}. \end{aligned} \tag{A.21}$$

This yields the tightness condition

$$(1-\tau_i^K)\frac{P_k k_i}{P} = q_i(\theta_i)(1-\beta_i)\frac{p_i f(k_i)/P - z}{D_i\,(r+s_i)} + \tau_i^T \tag{A.22}$$

Finally, we can derive the equilibrium condition for sectoral choice. From (A.1) and the Nash bargaining outcome $J_i^E(k_i) - J_i^U = \beta_i S_i(k_i)$, we have

$$\begin{aligned} rJ_i^U &= \theta_i q_i(\theta_i)\beta_i S_i(k_i) + \tau_i^L \\ &= \frac{\theta_i q_i(\theta_i)\,\beta_i\,[p_i f(k_i)/P - z]}{D_i} + \tau_i^L. \end{aligned}$$

So the optimal sectoral choice condition becomes

$$r(\epsilon_m - \epsilon_s) = \frac{\theta_m q_m(\theta_m)\,\beta_m\,[p_m f_m(k_m)/P - z]}{D_m} - \frac{\theta_s q_s(\theta_s)\,\beta_s\,[p_s f_s(k_s)/P - z]}{D_s} + (\tau_m^L - \tau_s^L). \tag{A.23}$$

Appendix B. Proof of Proposition 3

Proof. The current-value Hamiltonian associated with this problem is[14]

$$H = p_m Y_m/P + p_s Y_s/P - \sum_i m_i(t) P_k k_i(t)/P - \sum_i (r + s_i)\, v_i(t)\, P_k\, k_i(t)/P \tag{B.1}$$

$$- r\, L_s(t)\, \epsilon_s - r\,(1 - L_a - L_s(t))\epsilon_m$$

$$+ \sum_i \lambda_i^n(t)\,\big(m_i(t) - s_i n_i(t)\big) + \sum_i \lambda_i^Y(t)\,\big(m_i(t) f_i(k_i(t)) - s_i Y_i(t)\big)$$

$$+ z(u_m(t) + u_s(t)) \tag{B.2}$$

State variables: Y_i, n_i, controls: k_i, θ_i, L_s
Note that $u_i = L_i - n_i$, $v_i = \theta_i u_i = \theta_i(L_i - n_i)$, $m_i = \theta_i q_i u_i = \theta_i q_i(L_i - n_i)$.
$v_{\theta_i} = u_i$,
$q'(\theta_i) = -\eta \frac{q_i}{\theta_i}$,
$m_{\theta_i} = q_i u_i + \theta_i u_i q'(\theta_i) = u_i q_i(1 + \frac{\theta_i q_i'}{q_i}) = (1 - \eta) u_i q_i$.
The first order conditions with respect to k_i is:

$$H_{k_i} = 0: \quad [m_i + (r + s_i) v_i] P_k/P = \lambda_i^Y m_i f_i'(k_i) \tag{B.3}$$

$$\tag{B.4}$$

The co-state condition for Y_i:

$$\dot{\lambda}_i^Y = r\, \lambda_i^Y - H_{Y_i} \tag{B.5}$$

$$H_{Y_i} = p_i/P - \lambda_i^Y s_i. \tag{B.6}$$

Therefore, at steady state,

$$(r + s_i)\lambda_i^Y = \frac{p_i}{P} \tag{B.7}$$

Plug (B.7) into , and make use of $m_i + (r + s_i) v_i = m_i(1 + \frac{r+s_i}{q_i})$, we have the optimal

[14] Before deriving the first order conditions, we applied two mathematically equivalent transformations to the objective function. First, following Acemoglu and Shimer (1999), we rewrite the capital cost $I(t)$ into $\sum_i m_i(t) P_k k_i(t)/P - \sum_i (r + s_i)\, v_i(t)\, P_k\, k_i(t)/P$ in the integral. Second, we substitute $p_i = \frac{\partial C}{C_m}$. We can treat p_i "as if" it were exogenous without taking its derivative with respect to control variables due to the envelope theorem.

condition for the choice of k_i:

$$P_k = \frac{q_i p_i f_i'(k_i)}{(r + s_i + q_i)(r + s_i)} \tag{B.8}$$

FOC wrt θ_i.

$$H_{\theta_i} = m_{\theta_i}[-P_k k_i/P + \lambda_i^n + \lambda_i^Y f_i(k_i)] - (r + s_i)v_{\theta_i} P_k k_i/P = 0 \tag{B.9}$$

Denote $-P_k k_i/P + \lambda_i^n + \lambda_i^Y f_i(k_i) = \Phi_i$, the equation above can be simplified to

$$(1 - \eta)q_i\Phi_i = (r + s_i)P_k k_i/P \tag{B.10}$$

Co-state condition for n_i: at the steady state,

$$r\lambda_i^n = H_{n_i} = m_{n_i}[-P_k k_i/P + \lambda_i^n + \lambda_i^Y f_i(k_i)] - (r + s_i)v_{n_i} P_k k_i/P - s_i\lambda_i^n - z \tag{B.11}$$

$$= -\theta_i q_i \Phi_i + \theta_i(r + s_i)P_k k_i/P - s_i\lambda_i^n - z \tag{B.12}$$

where the last equality made use of $m_{n_i} = -\theta_i q_i$ and $v_{n_i} = -\theta_i$.

Hence

$$(r + s_i)\lambda_i^n = -z - \theta_i q_i \Phi + (r + s_i)P_k k_i/P \tag{B.13}$$

$$= -z - \theta_i q_i \Phi + \theta_i(1 - \eta)q_i\Phi_i \tag{B.14}$$

$$= -z - \eta\theta_i q_i \Phi_i \tag{B.15}$$

where the second equality made use of (B.10).

Use (B.15) to eliminate λ_i^n and plug into the definition of Φ_i and make use of (B.10), we have

$$\Phi_i = \frac{p_i f_i(k_i)/P - z}{D_i} \tag{B.16}$$

Plug it into (B.10) we have the tightness condition:

$$\frac{P_k k_i}{P} = (1 - \eta)q_i \frac{p_i f_i(k_i)/P - z}{(r + s_i)D_i} \tag{B.17}$$

FOC with respect to L_m

$$r\Delta\epsilon = \theta_m q_m \Phi_m - \theta_s q_s \Phi_s - \theta_m(r + s_m)P_k k_m/P + \theta_s(r + s_s)P_k k_s/P \tag{B.18}$$

where the equations above made use of

$$\frac{\partial m_m}{\partial L_m} = \theta_m q_m \tag{B.19}$$

$$\frac{\partial m_s}{\partial L_m} = -\theta_s q_s \tag{B.20}$$

$$\frac{\partial v_m}{\partial L_m} = \theta_m \tag{B.21}$$

$$\frac{v_s}{\partial L_m} = -\theta_s \tag{B.22}$$

$$\tag{B.23}$$

Substitute (B.10) into (B.18), we get the optimality condition for sectoral choice

$$\eta\theta_m q_m \Phi_m - \eta\theta_s q_s \Phi_s = r\Delta\epsilon \tag{B.24}$$

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PUBLICATIONS

Capital Holdup, Job Creation, and Skill Supply under Search Frictions

Working Paper No. WP/2026/093